Piggyback on TNCs for Electricity Services: Spatial Pricing and Synergetic Value

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***Abstract*— Major Transportation Network Companies (TNCs), including Uber and Lyft, are in the process of electrifying their fleets. These electrified fleets can provide electricity service in addition to transportation services. This paper examines the problem of optimal spatial pricing for a TNC operating a fleet of electric vehicles (EVs) that provide both transportation service and electricity service. When the system demand is spatially balanced, we establish a tight characterization of the optimal pricing and compensation policy, and the platform profit. Furthermore, we highlight the role electricity service may provide in reducing the transportation spatial imbalance and characterize the resulting synergetic value.**

I. Introduction

With the continuing decarbonization efforts in the electricity sector, the transportation sector has surpassed the electricity sector to become the largest source of U.S. greenhouse emission since 2016 [1]. As average vehicles are parked 95% of the time, prioritizing the electrification of vehicles that have a higher travel intensity (usually measured in vehicle-miles traveled (VMT)) can lead to a larger impact on emission reduction. Vehicles operated by Transportation Network Companies (TNCs) are among those with highest VMT [2]. Policies aiming to steer TNCs to electrify have emerged. Examples include California's Clean Miles Standard [3] and the Delhi (draft) Electric Vehicle Policy [4]. Meanwhile, major TNCs have set goals and experimented with a variety of programs to encourage electrification. For instance, Uber has a goal of delivering five million electric rides by 2019 while Lyft aims to deliver one billion electric rides by 2025 [5]. Given these trends, it has be projected that EVs will account for 80% of the shared mobility fleet by 2040 [6].

In addition to transportation service, electrified TNCs can provide other services to create new revenue streams and offset the upfront cost of the EVs. Electricity service is a natural candidate. The underlying technology for EVs to offer electricity service (e.g. vehicle-to-grid and vehicle-to-building) has been tested and piloted in various studies [7], [8]. New business models (e.g. demand charge reduction, EV roadside assistance and emergency power supply) that only require EVs to connect for a short period of time on demand have also been proposed or tested [9]–[11].

Spatial balancing is a key challenge in TNCs' (electrified or not) daily operation. It involves creating location-dependent incentives to nudge the supply and demand in order to minimize their spatial mismatch. Spatial mismatch is costly to TNCs because it results in higher waiting times (and therefore a higher cancellation rate) in some locations and idle drivers in other locations. Pricing and compensation policies are powerful tools in reducing spatial mismatch as spatial pricing can (to a large extent) shape the spatial distribution of demand while location-dependent compensation can incentivize drivers to relocate to the right locations. The optimal spatial pricing problem is to identify a location dependent pricing and compensation policy that maximizes the profit of the TNC (via minimizing the spatial mismatch).

This paper studies the spatial pricing problem in the context of shared electric mobility. We consider a TNC that provides transportation and electricity services by matching EVs to user requests. Offering transportation service requires an EV to move from a location to another, while offering electricity service requires an EV to park at a location for a certain time period. These two types of services have different location-dependent request rates and can have different location-dependent prices and compensation. Since the same set of EVs are used for both services, a strong coupling effect exists between offering them. The fundamental goal of this paper is to understand and quantify this coupling effect with a focus on spatial balancing. In particular, we wish to answer the following question: *whether and when does offering two services create synergetic value?*

In the process of answering this question, we make the following contributions. We develop a stylized model of a TNC providing transportation and electricity services over a network of locations. Within the model, the TNC's pricing and compensation policy is determined by solving a nonconvex optimization that maximizes the per period platform profit under incentive compatibility constraints and equilibrium flow constraints. We show that the optimal solution of this nonconvex program can be recovered from a simpler optimization, which is convex for a number of commonly used willingness to pay distributions. Focusing on uniform distributions, we develop a tight characterization of the optimal spatial pricing policy and closed form expressions for the optimal platform profit and the system equilibrium states. Furthermore, we identify conditions on the spatial demand patterns under which providing electricity service reduces transportation spatial imbalance and thus generates strictly positive synergetic value.

Literature. There is a growing literature addressing various challenges in the pricing and operation of TNCs. Among them, Banerjee et al. [12] develops a queueing theoretic model to study the optimal pricing problem in a TNC

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platform and analyzes the performance of dynamic and static pricing rules focusing on the temporal aspect of the problem. Bimkipis et al. [13], on the other hand, investigates the spatial pricing problem for a TNC operating over a network of locations. Several other related aspects are also investigated, see e.g. [14] for regulating TNCs, [15] for the role of self-scheduling, and [16] for relocation of strategic drivers.

The literature on using TNC EVs to provide electricity service is limited. The only work that we are aware of is Mamalis et al. [17], which extends queueing theoretic model of [12] to study the problem of designing driver incentives to maximize the platform revenue when two services are provided. No spatial aspect is considered in [17].

Our spatial pricing model is an extension of Bimkipis et al. [13]. Novel aspects of our model include introducing the electricity service in the spatial pricing context, characterizing the interaction between transportation and electricity services, and uncovering the demand-balancing role that may be played by offering electricity service.

II. Model

A. Network

Consider a network of locations $\{1,\ldots,n\}$. In this paper, we focus on the case of *equidistant networks*, where the distance between any two locations is the same and is normalized to one. Extensions to general networks will be studied in the future.

B. Time

We consider an infinite horizon discrete time setup. Within one time period, an EV can either fulfill a (transportation or electricity) service request or travel from one location to another. Since we are focusing on the spatial aspect of the problem, we will assume that the inputs are time-invariant and study the equilibrium outcomes of the model[1].

C. User Requests

The platform receives two types of requests: transportation service requests and electricity service requests. To fulfill a transportation service request, a matched EV will drive the user from one location to another as specified by the user. In the equidistant network case, this takes one period of time. To fulfill an electricity service request, a matched EV will park at a user-specified location for one period of time[2].

We model transportation requests by a tuple $(\mathbf{A}, \boldsymbol{\theta}, F(\cdot))$. Here $\theta_i \geq 0$, $i=1,\ldots,n$ is the potential request rate at location i. The $ij-$th entry of matrix $\mathbf{A}$, denoted by α_{ij}, is the fraction of users at location i who wish to go to location j. The matrix $\mathbf{A}$ is a stochastic matrix: $\alpha_{ij} \geq 0$ for all i and j and $\sum_{j=1}^{n} \alpha_{ij} = 1$. It can also be viewed as a weighted adjacency matrix associated with the network, where there is a directed edge from i to j if $\alpha_{ij} > 0$. We assume the graph defined by adjacency matrix $\mathbf{A}$ is strongly connected. The rate of potential requests for trips from i to j is $\theta_i \alpha_{ij}$. If the price of the platform charges for using the service is p, then the actual request rate for such trips is $(1-F(p))\theta_i\alpha_{ij}$, where $F(\cdot)$, with support $[0,u]$, models the empirical cumulative distribution of the willingness to pay for using the transportation service.

The electricity service requests are modeled by a pair $(\widetilde{\boldsymbol{\theta}}, \widetilde{F}(\cdot))$. Here $\widetilde{\theta}_i \geq 0$, $i=1,\ldots,n$ is the potential request rate at location i. If the price of the platform charges for using the service is $\widetilde{p}$, then the actual request rate for such service is $(1-\widetilde{F}(\widetilde{p}))\widetilde{\theta}_i$, where $\widetilde{F}(\cdot)$, with support $[0,\widetilde{u}]$, models the empirical cumulative distribution of the willingness to pay for using the electricity service.

D. EV Drivers

Due to the timeliness requirement of the service, we assume that user requests at location i can only be served by EVs located at i in the same time period. Under this assumption, drivers may move from location i to location j for two reasons: (a) fulfilling a transportation service trip, or (b) relocating to another location with a higher potential earning on their own.

Upon completing a service or relocating to another location, drivers may exit the platform with probability $(1-\beta)$, where $\beta \in (0,1)$ captures the fact that in expectation a driver only provides service for a limited time period. We refer to this time period as a driver's lifetime. We assume at any location there is an infinite supply of drivers. Each driver has an outside option with expected lifetime earning $w > 0$ such that a driver will choose to enter the platform only if the expected earning of entering is no smaller than w.

E. Pricing

The platform determines its pricing and compensation policy for transportation service $\{p_i, c_i\}_{i=1}^{n}$ and electricity service $\{\widetilde{p}_i, \widetilde{c}_i\}_{i=1}^{n}$. Here $p_i \geq 0$ is the price the platform charges the users of transportation service *originating*[3] from location i and $c_i \geq 0$ is the compensation the platform pays the drivers. Similarly, $\widetilde{p}_i \geq 0$ and $\widetilde{c}_i \geq 0$ denote the price and compensation the platform sets for electricity service at location i.

Denote the actual request arrival rates for transportation and electricity services at location i by λ_i and $\widetilde{\lambda}_i$, respectively. Given the pricing policy, the actual request arrival rate for transportation service from location i to location j is $(1-F(p_i))\theta_i\alpha_{ij}$, and we have

$$\lambda_i = (1-F(p_i))\theta_i, \quad \forall i. \tag{1}$$

Similarly, the actual request arrival rate for electricity service at location i is

$$\widetilde{\lambda}_i = (1-\widetilde{F}(\widetilde{p}_i))\widetilde{\theta}_i, \quad \forall i. \tag{2}$$

[1]Our results can also be interpreted as the system is operating at the equilibrium for a single time period.

[2]Our model may be applied to a variety of electricity services mentioned in Section I provided that the following requirements hold: (a) the duration of the service is comparable to the typical duration for a transportation service, which is on the order of 15 minutes, and (b) the energy (kWh) requirement for providing is low or modest. Demand charge reduction and emergency roadside battery recharge are two such examples.

[3]Given the equidistant network assumption, we focus on origin-only pricing for transportation service in this paper. See [13] for a detailed comparison between origin-only pricing and origin-destination pricing.

F. Matching

The same pool of EVs are used to fulfill the two types of service requests. We assume that drivers do not have an option to refuse a request. Instead, they have the option to exit the platform at the end of each period.

An EV at location i is deemed available if the driver maintains an active communication link with the platform through the mobile application and if the EV's battery has enough energy to provide any type of service the platform offers.

The platform distinguishes the EVs only by their location at the beginning of each time period. At location i, when a new request arrives (either for providing transportation service or electricity service), an EV located at the same location gets matched to the request uniformly at random, given that there is a sufficient number of available EVs at i. When the number of requests at a location and time exceeds the number of EVs available at that location, *priority is given to transportation service*.[4] The excess requests are not matched and simply exit the system.

Denote the mass of drivers at location i at the beginning of a period by x_i. Under this matching policy, the number of served transportation requests at location i in a given period (throughput) is

$$Q_i(x_i, \lambda_i) := \min\{x_i, \lambda_i\}, \tag{3}$$

and the number of served electricity requests at location i in a given period is

$$\widetilde{Q}_i(x_i, \lambda_i, \widetilde{\lambda}_i) := \min\left\{(x_i - \lambda_i)_+, \widetilde{\lambda}_i\right\}. \tag{4}$$

We denote the probability that a driver at i gets matched to a transportation service request by $\rho_i(x_i, \lambda_i)$ and the probability that a driver at i gets matched to an electricity service request by $\widetilde{\rho}_i(x_i, \lambda_i, \widetilde{\lambda}_i)$. Under the transportation priority rule, it is easy to see that

$$\rho_i(x_i, \lambda_i) = \min\left\{(\lambda_i/x_i), 1\right\}, \tag{5}$$

$$\widetilde{\rho}_i(x_i, \lambda_i, \widetilde{\lambda}_i) = \mathbb{1}\{x_i > \lambda_i\} \min\left\{\frac{\widetilde{\lambda}_i}{x_i}, \frac{x_i - \lambda_i}{x_i}, 1\right\}. \tag{6}$$

G. Platform Profit

The goal of the platform is to maximize its (per period) profit under the equilibrium outcome induced by the pricing and compensation policy $\{p_i, c_i, \widetilde{p}_i, \widetilde{c}_i\}_{i=1}^n$. In particular, the profit of the platform has the following form:

$$\sum_i (p_i - c_i) Q_i(x_i, \lambda_i) + (\widetilde{p}_i - \widetilde{c}_i)\widetilde{Q}_i(x_i, \lambda_i, \widetilde{\lambda}_i),$$

where λ_i, $\widetilde{\lambda}_i$, x_i are the induced equilibrium values.The precise form of this dependence is outlined in the next section.

[4]It will become clear later (see the proof of Lemma 1) that our results also apply to the no priority case and to the case where the priority is given to electricity service.

III. Platform Optimization

A. Equilibrium

Denote the mass of drivers entering at location i at the beginning of a period by δ_i, and the mass of drivers at location i who decide to relocate to location j upon not getting matched to a service request by y_{ij}.

At equilibrium, by flow conservation,

$$x_i = \beta \left[\sum_j \alpha_{ji} Q_j(x_j, \lambda_j) + \widetilde{Q}_i(x_i, \lambda_i, \widetilde{\lambda}_i) + \sum_j y_{ji}\right] + \delta_i, \tag{7}$$

for all i. The first term in the bracket is the mass of drivers who move to location i while fulfilling transportation service. The second term in the bracket is the mass of drivers who stay at location i for fulfilling electricity service. The third term in the bracket is the mass of drivers who move to location i on their own for a higher expected earning. The last term is the mass of drivers who enter the platform at location i.

Meanwhile, the total mass of drivers who are not matched satisfies the following identity

$$\sum_j y_{ij} = \left[x_i - \lambda_i - \widetilde{\lambda}_i\right]_+, \quad \forall i, \tag{8}$$

where $[z]_+ := \max(z, 0)$. Here $\sum_{j \neq i} y_{ij}$ is the total mass of unmatched drivers moving to a different location j and y_{ii} is the mass of unmatched drivers who stay at i. We refer to (7) and (8) as *equilibrium flow constraints*.

Denote the expected future earning for a driver at each location i by V_i. At equilibrium,

$$\begin{aligned} V_i =& \rho_i(x_i, \lambda_i)\big(c_i + \sum_j \alpha_{ij}\beta V_j\big) + \widetilde{\rho}_i(x_i, \lambda_i, \widetilde{\lambda}_i)\left(\widetilde{c}_i + \beta V_i\right) \\ &+ \Big(1 - \rho_i(x_i, \lambda_i) - \widetilde{\rho}_i(x_i, \lambda_i, \widetilde{\lambda}_i)\Big)\, \beta \max_j V_j, \end{aligned} \tag{9}$$

for all i. The first term is the expected earning from offering transportation service. The second term is the expected earning from offering electricity service. The last term is the expected earning for the driver to be unmatched and to relocate to the location with the maximum expected earning.

Since the outside earning of the driver is w, and since there are an infinite number of potential drivers, we have at the equilibrium $V_i \leq w$. On the other hand, for a driver to enter the platform, he must earn at least w. Thus

$$V_i = \max_j V_j = w, \quad \text{for all } i \text{ such that } \delta_i + \sum_j y_{ji} > 0, \tag{10}$$

where locations such that $\delta_i + \sum_j y_{ji} > 0$ are those with a positive mass of drivers entering or relocating to. We refer to (9) and (10) as drivers' *incentive compatibility constraints*.

We formally define the notion of equilibrium as follows.

Definition 1 (Equilibrium)**.** *Given a pricing and compensation policy $\{p_i, \widetilde{p}_i, c_i, \widetilde{c}_i\}_{i=1}^n$, an equilibrium is a tuple $\{\lambda_i, \widetilde{\lambda}_i, x_i, y_{ij}, \delta_i\}_{i,j=1}^n$ such that $\lambda_i, \widetilde{\lambda}_i, x_i, \delta_i, y_{ij} \geq 0$ for all $i, j = 1, \ldots, n$, satisfying users' incentive compatibility* *constraints* (1) *and* (2)*, equilibrium flow constraints* (7) *and* (8)*, and drivers' incentive compatibility constraints* (9) *and* (10).

B. Profit Maximization

The platform's goal is to maximize its equilibrium profit rate by setting a pricing and compensation policy:

$$\begin{aligned} \max \quad & \sum_i (p_i - c_i) Q_i(x_i, \lambda_i) + (\widetilde{p}_i - \widetilde{c}_i) \widetilde{Q}_i(x_i, \lambda_i, \widetilde{\lambda}_i) \\ \text{s.t.} \quad & \{\lambda_i, \widetilde{\lambda}_i, x_i, y_{ij}, \delta_i\}_{i,j=1}^n \text{ is an equilibrium} \\ & \text{under } \{p_i, \widetilde{p}_i, c_i, \widetilde{c}_i\}_{i=1}^n, \end{aligned} \tag{11}$$

where the decision variables include the prices and compensation $p_i, \widetilde{p}_i, c_i, \widetilde{c}_i \geq 0$, and the equilibrium states $\lambda_i, \widetilde{\lambda}_i, x_i, y_{ij}, \delta_i,\ i, j \in \{1, \ldots, n\}$. This is a challenging optimization due to nonconvex equilibrium constraints. In fact, it is not even immediately clear whether an equilibrium (and therefore a feasible solution of problem (11)) exists. We show this is indeed the case:

Proposition 1. *There exists a feasible solution to* (11)*. Furthermore, there exists an equilibrium under every non-negative* $\{p_i, \widetilde{p}_i, c_i, \widetilde{c}_i\}_{i=1}^n$*, provided* $\theta_i > 0$ *for all* $i \in \{1, \ldots, n\}$.

The first part of Proposition 1 establishes that regardless of the values of exogenous parameters, problem (11) always has a feasible solution (and therefore an optimal solution). The second part guarantees the existence of an equilibrium for any given pricing and compensation policy under additional conditions on the transportation demand patterns. Proposition 1 does not address the uniqueness of the equilibrium. In fact, there may be pricing and compensation policies under which the equilibrium states and the platform profit are not unique. However, it will become clear later that this is not the case under the optimal pricing and compensation policy.

Instead of directly tackling the nonconvex problem (11), we construct a simpler problem (12) and then formally establish the connection between these two problems:

$$\max \quad \sum_i p_i \lambda_i + \widetilde{p}_i \widetilde{\lambda}_i - w \sum_i \delta_i \tag{12a}$$

$$\text{s.t.} \quad \lambda_i = (1 - F(p_i))\theta_i, \quad \forall i, \tag{12b}$$

$$\widetilde{\lambda}_i = (1 - \widetilde{F}(\widetilde{p}_i))\widetilde{\theta}_i, \quad \forall i, \tag{12c}$$

$$x_i = \beta \left[\sum_j \alpha_{ji} \lambda_j + \widetilde{\lambda}_i + \sum_j y_{ji} \right] + \delta_i, \ \forall i, \tag{12d}$$

$$\sum_j y_{ij} = x_i - \lambda_i - \widetilde{\lambda}_i, \quad \forall i, \tag{12e}$$

$$p_i, \widetilde{p}_i, x_i, y_{ij}, \delta_i \geq 0, \quad \forall i, j, \tag{12f}$$

where the decision variables include the prices $\{p_i, \widetilde{p}_i\}_{i=1}^n$ and the equilibrium states $\{\lambda_i, \widetilde{\lambda}_i, x_i, \delta_i, y_{ij}\}_{i,j=1}^n$. Comparing (11) with (12), we notice the following differences: (a) the nonlinear equality flow constraints are replaced with linear ones, and (b) decision variables for compensation $\{c_i, \widetilde{c}_i\}_{i=1}^n$ no longer appear in the objective function, and (c) the drivers' incentive compatibility constraints are relaxed. Despite these differences, we can recover the optimal solution of (11) given an optimal solution of (12):

Lemma 1. *Suppose* $F(\cdot)$ *and* $\widetilde{F}(\cdot)$ *are absolutely continuous distributions that are strictly increasing over their supports. Suppose* $(1-\beta)w < \min\{u, \widetilde{u}\}$*. Let an optimal solution of* (12) *be denoted by* $\{p_i^\star, \widetilde{p}_i^\star, \lambda_i^\star, \widetilde{\lambda}_i^\star, x_i^\star, \delta_i^\star, y_{ij}^\star\}_{i,j=1}^n$*. Then there exists a compensation policy* $\{c_i^\star, \widetilde{c}_i^\star\}_{i=1}^n$ *such that* $\{p_i^\star, \widetilde{p}_i^\star, c_i^\star, \widetilde{c}_i^\star, \lambda_i^\star, \widetilde{\lambda}_i^\star, x_i^\star, y_{ij}^\star, \delta_i^\star\}_{i,j=1}^n$ *is an optimal solution of* (11).

The additional condition $(1-\beta)w < \min\{u, \widetilde{u}\}$ is imposed so we can focus on the interesting parameter region of the problem. In fact, this is precisely the condition such that drivers' outside option is not too high and they are willing to provide any service (transportation or electricity) through the platform. To see this, given w is the drivers' lifetime earning of the outside option, the minimum amount of per period compensation that the platform provides to a driver is $(1-\beta)w$ as $\sum_t \beta^t (1-\beta) w = w$. If this condition does not hold, at least one service cannot provide sufficient compensation to drivers, and thus, is inactive. Therefore, the maximum profit the platform can extract per request is $u - (1-\beta)w$ for transportation service, and $\widetilde{u} - (1-\beta)w$ for electricity service. For convenience, denote

$$\pi := u - (1-\beta)w, \quad \widetilde{\pi} := \widetilde{u} - (1-\beta)w.$$

If we focus on the case $\pi, \widetilde{\pi} > 0$, Lemma 1 establishes that it suffices to work with problem (12). While this optimization is still nonconvex for general distributions $F(\cdot)$ and $\widetilde{F}(\cdot)$, it is a convex optimization for several commonly used distributions (e.g. uniform and exponential). For simplicity, we work with uniform distributions for the rest of the paper. That is, the willingness to pay for transportation service follows a uniform distribution on $[0, u]$, and the willingness to pay for electricity service follows a uniform distribution on $[0, \widetilde{u}]$. In this case, (12) reduces to a convex quadratic program:

$$\max \quad \sum_i p_i \lambda_i + \widetilde{p}_i \widetilde{\lambda}_i - w \sum_i \delta_i \tag{13a}$$

$$\text{s.t.} \quad \lambda_i = \theta_i \left(1 - p_i / u\right), \quad \forall i, \tag{13b}$$

$$\widetilde{\lambda}_i = \widetilde{\theta}_i \left(1 - \widetilde{p}_i / \widetilde{u}\right), \quad \forall i, \tag{13c}$$

$$\delta_i = \sum_j (y_{ij} - \beta y_{ji}) + (\lambda_i - \beta \sum_j \alpha_{ji} \lambda_j) + (1-\beta)\widetilde{\lambda}_i, \quad \forall i, \tag{13d}$$

$$p_i, \widetilde{p}_i, y_{ij}, \delta_i \geq 0, \quad \forall i, j, \tag{13e}$$

$$p_i \leq u, \widetilde{p}_i \leq \widetilde{u}, \quad \forall i, \tag{13f}$$

where the decision variables are $\{p_i, \widetilde{p}_i, \lambda_i, \widetilde{\lambda}_i, y_{ij}, \delta_i\}_{i,j=1}^n$.

IV. Spatial Pricing and Synergetic Value

A. General Demand

The goal of this paper is to understand the interaction between providing transportation service and electricity service through the same TNC platform. From the TNC's perspective, this amounts to characterizing and comparing its optimal profits when it provides individual services versus when it jointly provides two services.

To this end, denote the optimal profit of the platform, i.e., the optimal value of problem (13), as a function of the demand patterns by $\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$. In general, we can write

$$\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}) = \Pi^\star(\boldsymbol{\theta}, \mathbf{0}) + \Pi^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}}) + \Delta,$$

where the term on the left hand side is the profit for jointly providing two services, and the first two terms on the right hand side are the profits for providing only transportation service and only electricity service. The difference Δ captures how the interaction between providing two services impacts the platform's profit. Thus the first step for understanding this interaction is to characterize the sign of Δ.

Lemma 2. *The synergetic value is nonnegative, i.e.,* $\Delta \geq 0$.

Lemma 2 states that jointly offering the two services is no worse than individually offering these services with designated sets of EVs. We can compare the effect of offering electricity service in addition to the existing transportation service with simply scaling up the number of requests:

Lemma 3. *The optimal platform profit* $\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$ *is homogenous of degree* 1*, i.e.,* $\Pi^\star(t\boldsymbol{\theta}, t\widetilde{\boldsymbol{\theta}}) = t\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$ *for any* $t > 0$.

By Lemma 3, increasing the request rates uniformly generates a constant rate of return. Therefore, Lemma 2 indeed suggests a potential synergy in providing two services that is beyond the effect of simply scaling up the system demand.

B. Balanced Demand

To provide a tighter characterization of the synergetic value Δ, we consider the following condition on the demand patterns.

Definition 2 (Balanced Demand)**.** *The platform's system demand* $(\mathbf{A}, \boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$ *is deemed balanced if*

$$\frac{\pi}{u}(\mathbf{I} - \beta\mathbf{A}^\top)\boldsymbol{\theta} + \frac{\widetilde{\pi}}{\widetilde{u}}(1-\beta)\widetilde{\boldsymbol{\theta}} \geq \mathbf{0}. \tag{14}$$

The platform's transportation demand $(\mathbf{A}, \boldsymbol{\theta})$ *is deemed balanced if* (14) *holds with* $\widetilde{\theta} \equiv 0$*, i.e.,*

$$(\mathbf{I} - \beta\mathbf{A}^\top)\boldsymbol{\theta} \geq 0. \tag{15}$$

When $u = \widetilde{u}$ (and therefore $\pi = \widetilde{\pi}$), the balanced demand condition can be written as

$$\boldsymbol{\theta} + \widetilde{\boldsymbol{\theta}} \geq \beta\mathbf{A}^\top\boldsymbol{\theta} + \beta\widetilde{\boldsymbol{\theta}},$$

and is easy to interpret. Consider two consecutive time periods. The left hand side of the inequality is the total demand in the second time period, while the right hand side is the vector representing the mass of drivers who stayed in the platform after providing a service. If this condition does not hold for certain location i, then there is some driver who decided to stay in the platform after providing the service and will end up unmatched in the second time period. Note that if $\boldsymbol{\theta} = \mathbf{0}$, this condition always holds. This is because transportation demand is the source of spatial imbalance as it requires driver to move between locations.

When the demand is balanced, we can provide a complete characterization of the solution of (13):

Theorem 1. *Suppose* (14) *holds and* $\pi, \widetilde{\pi} > 0$.

1) *The optimal profit is*
$$\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}) = \frac{\pi^2}{4u}\mathbf{1}^\top\boldsymbol{\theta} + \frac{\widetilde{\pi}^2}{4\widetilde{u}}\mathbf{1}^\top\widetilde{\boldsymbol{\theta}}.$$
2) *The platform maximizes its profit by setting identical prices across locations:*
$$p_i^\star = \frac{u}{2} + \frac{(1-\beta)w}{2}, \quad \widetilde{p}_i^\star = \frac{\widetilde{u}}{2} + \frac{(1-\beta)w}{2}, \quad \forall i.$$
The corresponding compensation is
$$c_i^\star = \Big(1 + \frac{\widetilde{\pi}u\widetilde{\theta}_i}{\pi\widetilde{u}\theta_i}\Big)\frac{(1-\beta)w}{2}, \ \widetilde{c}_i^\star = \Big(1 + \frac{\pi\widetilde{u}\theta_i}{\widetilde{\pi}u\widetilde{\theta}_i}\Big)\frac{(1-\beta)w}{2}.$$
3) *The induced system equilibrium states are*
$$\lambda_i^\star = \frac{\pi}{2u}\theta_i, \quad \widetilde{\lambda}_i^\star = \frac{\widetilde{\pi}}{2\widetilde{u}}\widetilde{\theta}_i, \quad \forall i,$$
$$x_i^\star = \frac{\pi}{2u}\theta_i + \frac{\widetilde{\pi}}{2\widetilde{u}}\widetilde{\theta}_i, \quad \forall i,$$
$$\delta_i^\star = \frac{\pi}{2u}\Big(\theta_i - \beta\sum_j \alpha_{ji}\theta_j\Big) + (1-\beta)\frac{\widetilde{\pi}}{2\widetilde{u}}\widetilde{\theta}_i, \quad \forall i,$$
and $y_{ij}^\star = 0$ *for all* i, j.

Theorem 1 suggests that when the demand is balanced, the optimal prices are location independent. The compensation is such that the expected platform earning $V_i = w$ for all i (see the proof of Lemma 1). As a result, no drivers will relocate on their own ($\mathbf{Y}^\star = \mathbf{0}$), and there is no idle vehicle as every EVs at any location will be matched to a request at every period in the equilibrium. In a sense, this is the ideal scenario for the TNC. This is made precise as follows.

Corollary 1. *Fix* $\boldsymbol{\theta}$ *and* $\widetilde{\boldsymbol{\theta}}$*. The platform's optimal profit as a function of* $\mathbf{A}$ *achieves its maximal when* $\mathbf{A}$ *is such that the platform's demand is balanced.*

To characterize the synergetic value under the balanced demand condition, we note that the solutions and optimal profits under the transportation only and the electricity only setups can be recovered as special cases of Theorem 1.

Corollary 2. *Suppose* $\pi, \widetilde{\pi} > 0$.

1) *Transportation only case: If* $\boldsymbol{\theta} > \mathbf{0}$*,* $\widetilde{\boldsymbol{\theta}} = \mathbf{0}$*, and the transportation demand is balanced (i.e.* (15) *holds), then all statements in Theorem 1 holds when we set* $\widetilde{\boldsymbol{\theta}} = \mathbf{0}$.
2) *Electricity only case: If* $\boldsymbol{\theta} = \mathbf{0}$ *and* $\widetilde{\boldsymbol{\theta}} > \mathbf{0}$*, all statements in Theorem 1 holds when we set* $\boldsymbol{\theta} = \mathbf{0}$.

Since $\widetilde{\boldsymbol{\theta}} \geq \mathbf{0}$, the transportation demand balance condition (15) is more stringent than the demand balance condition (14). When the transportation demand is already balanced, there is no spatial imbalance that can be reduced by the electricity demand, and therefore one expects no synergy

Fig. 1: Three node network based on Uber SF data

between the two services. When we have imbalanced transportation demand, the electricity service reduces the resulting spatial imbalance and generates strictly positive synergetic value. We summarize these observations as follows:

Corollary 3. *Suppose* $\pi, \widetilde{\pi} > 0$ *and* $\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}} > \mathbf{0}$.

1) *If* (15) *(and therefore* (14)*) holds, then* $\Delta = 0$.
2) *If* (14) *holds but* (15) *does not hold, then* $\Delta > 0$.

V. Case Study

In this section, we empirically compare the profits of the TNC when the two services are jointly offered versus when they are individually offered. We consider a three node network and take each time period to be 20 minutes. This is sufficient to complete an average TNC trip [14] and to complete a demand charge reduction service session [9]. Based on the discussion in [9], the revenue generated for electricity service can be an order of magnitude higher than the average transportation service. However, this revenue is severely discounted by the high upfront cost of required additional hardware (e.g. bidirectional charger), the concern on battery degradation, and the difficulty in forecasting the time period when the monthly peak of the user occurs. Thus we set $u = 1$ and $\widetilde{u} = 1.5$. We set $w = 1$ so $\pi, \widetilde{\pi} > 0$ holds.

The transportation demand pattern is based on the Uber Newsroom's San Francisco data[5]. In particular, we pick three nodes with largest request rates and in close proximity (Fig. 1). We use the line width data to obtain the origin-destination matrix and use the circle radius data to obtain the transportation demand:

$$\mathbf{A} = \begin{bmatrix} 0.2736 & 0.2760 & 0.4504 \\ 0.2188 & 0.2616 & 0.5196 \\ 0.2244 & 0.3634 & 0.4122 \end{bmatrix}, \quad \boldsymbol{\theta}^{\text{bal}} = \begin{bmatrix} 0.2985 \\ 0.3365 \\ 0.3650 \end{bmatrix},$$

where we have normalized $\mathbf{A}$ so each row sums up to 1 (ignoring lines connecting to other nodes not considered) and normalized $\boldsymbol{\theta}^{\text{bal}}$ so that the total transportation demand is 1. Since this data correspond to the *average* Uber request rate for a month (July 2014), it is mostly spatially balanced. For instance, we can check the transportation balanced demand condition for $\beta \in \{0.6, 0.9\}$

$$(\mathbf{I} - 0.6\mathbf{A}^\top)\boldsymbol{\theta} = \begin{bmatrix} 0.1562 \\ 0.1547 \\ 0.0892 \end{bmatrix}, \ (\mathbf{I} - 0.9\mathbf{A}^\top)\boldsymbol{\theta} = \begin{bmatrix} 0.0850 \\ 0.0637 \\ -0.0488 \end{bmatrix}.$$

Thus even with very large β, the level of imbalancedness of the transportation demand is very small. As such, by Corollary 3, we expect the synergetic value to be 0 for small β values and close to 0 for larger ones.

We set $\widetilde{\boldsymbol{\theta}}^{\text{bal}} = \mathbf{1}/3$ and perform two set of experiments: (a) varying β between $[0, 1]$, and (b) fix $\beta = 0.6$ and scale $\widetilde{\boldsymbol{\theta}}$ by k where k varies between $[0, 3]$.

The profit for jointly offering two services $(\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}))$ and the sum of profits for individually offering the services $(\Pi^\star(\boldsymbol{\theta}, \mathbf{0}) + \Pi^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}}))$ are compared in Fig 2. We also plot the profit for only offering transportation service. In addition to the profits, we label the parameter regions based on whether the transportation (labelled T) demand is balanced (labelled B) or imbalanced (labelled IB) per (15), and whether the system (labelled S) demand is balanced (i.e., whether (14) holds). The maximum percentage profit improvement $(\Delta/(\Pi^\star(\boldsymbol{\theta}, \mathbf{0}) + \Pi^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}})))$ is 0.26% for experiment (a) and 0% for experiment (b) (since the transportation demand is already balanced with $\beta = 0.6$).

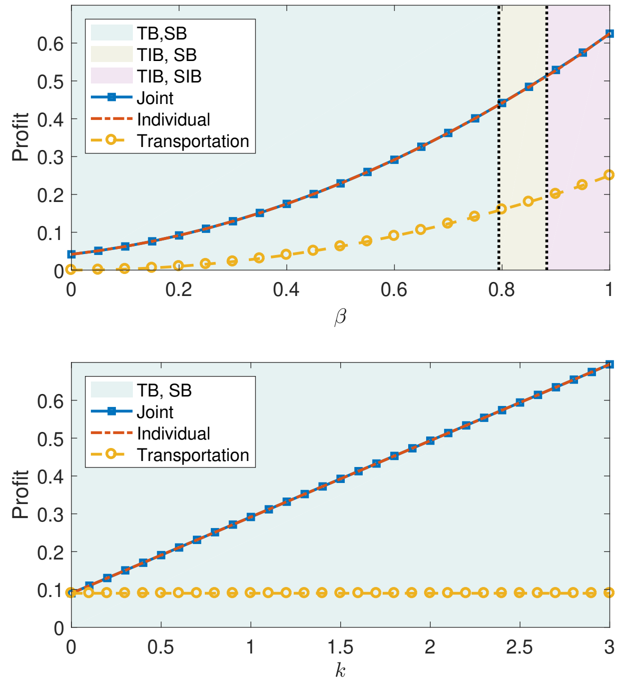

Fig. 2: Profit comparison for a (mostly) balanced transportation demand pattern

We also consider another profile of transportation demand where the spatial imbalance is more significant and a corresponding profile of electricity demand that will help reduce this spatial imbalance

$$\boldsymbol{\theta}^{\text{unb}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top, \quad \widetilde{\boldsymbol{\theta}}^{\text{unb}} = \begin{bmatrix} 0.0 & 0.5 & 0.5 \end{bmatrix}^\top.$$

To check the level of spatial imbalance in the transportation

[5]https://newsroom.uber.com/wp-content/uploads/2014/07/uber_sf_connectome_.html

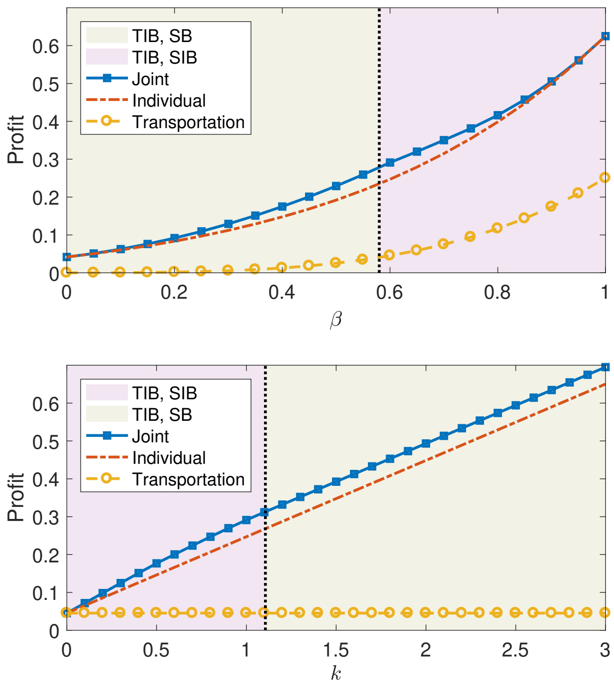

Fig. 3: Profit comparison for an imbalanced transportation demand pattern

demand, for $\beta \in \{0.6, 0.9\}$, we compute

$$(\mathbf{I}{-}0.6\mathbf{A}^\top)\boldsymbol{\theta} = \begin{bmatrix} 0.8358 \\ -0.1656 \\ -0.2702 \end{bmatrix}, \; (\mathbf{I}{-}0.9\mathbf{A}^\top)\boldsymbol{\theta} = \begin{bmatrix} 0.7537 \\ -0.2484 \\ -0.4054 \end{bmatrix}.$$

The same two sets of experiments are performed for these demand patterns. The profit comparisons are shown in Fig 3. The maximum percentage profit improvement is 19.35% for experiment (a) and 20.95% for experiment (b).

VI. Conclusion

This paper studies the spatial pricing problem for a TNC offering both transportation and electricity services. It is formulated as a profit maximization problem under incentive compatibility and equilibrium flow constraints. We show that we can recover the solution of this nonconvex optimization via solving a simpler problem, which is convex for several common willingness to pay distributions. With uniform willingness to pay distributions and for general demand, we show that offering electricity service will generate a nonnegative synergetic value. Furthermore, when the transportation demand is spatially unbalanced, we show that offering electricity service can reduce such imbalance, and therefore generate a strictly positive synergetic value.

Appendix

Proof Sketch of Proposition 1. To prove the first part of the proposition, notice $p_i = u$, $\widetilde{p}_i = \widetilde{u}$, $c_i = \widetilde{c}_i = 0$, $x_i = \lambda_i = \widetilde{\lambda}_i = \delta_i = y_{ij} = 0$ for all i and j is a feasible solution.

The proof for the second part is omitted due to page limit. It amounts to constructing a game with agents who have compact and convex strategy spaces and (quasi-)concave and (upper-semi)continuous payoff functions, then formally connecting the game's pure strategy Nash equilibrium with the equilibrium as defined in Definition 1. See [13] for a similar construction. □

Proof of Lemma 1. We first prove a number of claims.

(a) There exists a solution to the optimization such that $x_i^\star \geq \lambda_i^\star + \widetilde{\lambda}_i^\star$, $\forall i$. By contradiction, we assume that for all solutions $x_i^\star < \lambda_i^\star + \widetilde{\lambda}_i^\star$ for some i.

- Case I: $x_i^\star \geq \lambda_i^\star$, $x_i^\star - \lambda_i^\star < \widetilde{\lambda}_i^\star$. Increasing $\widetilde{p}_i$ weakly increases the profit.
- Case II: $x_i^\star < \lambda_i^\star$. Increasing p_i weakly increases the profit.

(b) Solutions satisfying (a) also satisfies $(1-\beta)\sum_i x_i^\star = \sum_i \delta_i^\star$. Using (a), we can simplify (7) to

$$x_i = \beta\Big[\sum_j \alpha_{ji}\lambda_j + \widetilde{\lambda}_i + \sum_j y_{ji}\Big] + \delta_i, \quad \forall i,$$

and simplify (8) to

$$\sum_j y_{ij} = x_i - \lambda_i - \widetilde{\lambda}_i, \quad \forall i.$$

Summing up both sides of the equations, we get

$$\begin{aligned}\sum_i x_i &= \beta\sum_i(\lambda_i + \widetilde{\lambda}_i) + \beta\sum_i\sum_j y_{ji} + \sum_i \delta_i \\ &= \beta\sum_i(\lambda_i + \widetilde{\lambda}_i) + \beta\sum_i(x_i - \lambda_i - \widetilde{\lambda}_i) + \sum_i \delta_i,\end{aligned}$$

and therefore $(1-\beta)\sum_i x_i = \sum_i \delta_i$.

(c) Among solutions satisfying (a), if $\lambda_i^\star, \widetilde{\lambda}_i^\star > 0$, then there exist solutions such that $\sum_i c_i^\star\lambda_i^\star + \widetilde{c}_i^\star\widetilde{\lambda}_i^\star = w\sum_i \delta_i^\star$.

By (a), (9) simplifies to

$$x_iV_i = \lambda_i\Big[c_i + \sum_j \alpha_{ij}\beta V_j\Big] + \widetilde{\lambda}_i(\widetilde{c}_i + \beta V_i) + (x_i - \lambda_i - \widetilde{\lambda}_i)\beta w,$$

for all i. Summing over i, we have

$$\sum_i \Big[x_i - \beta(\lambda_i + \widetilde{\lambda}_i)\Big] V_i = \sum_i \lambda_i c_i + \widetilde{\lambda}_i\widetilde{c}_i + (x_i - \lambda_i - \widetilde{\lambda}_i)\beta w$$

Note that if $V_i = w$ for all i, using (b) we have

$$\sum_i \lambda_i c_i + \widetilde{\lambda}_i\widetilde{c}_i = (1-\beta)w\sum_i x_i = w\sum_i \delta_i.$$

Thus the proof is completed if we can construct $\{c_i, \widetilde{c}_i\}_{i=1}^n$ such that $V_i = w$ for all i. Consider

$$c_i = \frac{x_i}{2\lambda_i}(1-\beta)w, \quad \widetilde{c}_i = \frac{x_i}{2\widetilde{\lambda}_i}(1-\beta)w, \quad \forall i. \tag{16}$$

With this compensation policy and using (a), the expected one period earning of a driver is $\frac{\lambda_i}{x_i}c_i + \frac{\widetilde{\lambda}_i}{x_i}\widetilde{c}_i = (1-\beta)w$, and the expected lifetime earning of the driver is $\sum_t \beta^t(1-\beta)w = w$. Thus $V_i = w$ $\forall i$ with this compensation policy.

From (a), there is no loss of optimality by replacing the flow constraints by (12d) and (12e) and setting $Q_i(x_i, \lambda_i) = x_i$ and $\widetilde{Q}_i(x_i, \lambda_i, \widetilde{\lambda}_i) = \widetilde{\lambda}_i$ in the objective. From (c), the objective function of (11) can be replaced with the objective function of (12). Lastly, (16) provides a way to reconstruct

$\{c_i, \widetilde{c}_i\}_{i=1}^n$ given a solution of (12) such that the drivers' incentive compatibility constraints are satisfied. $\square$

Proof of Lemma 2. Denote $\mathbf{x} = \{p_i, \widetilde{p}_i, \lambda_i, \widetilde{\lambda}_i, y_{ij}, \delta_i\}_{i,j=1}^n$. Let an optimal solution of (13) with demand pattern $(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$ be denoted by $\mathbf{x}^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$, and let the profit of the platform as a function of $\mathbf{x}$, $\boldsymbol{\theta}$, $\widetilde{\boldsymbol{\theta}}$ be denoted by $\Pi(\mathbf{x}; \boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$. Without loss of generality, we can set $\widetilde{p}_i = \widetilde{u}$ and $\widetilde{\lambda}_i = 0$ for all i in $\mathbf{x}^\star(\boldsymbol{\theta}, \mathbf{0})$. Similarly, we can set $p_i = u$ and $\lambda_i = 0$ for all i in $\mathbf{x}^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}})$. Consider $\mathbf{x}^\star(\boldsymbol{\theta}, \mathbf{0}) + \mathbf{x}^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}})$. It is easy to check that it is a feasible solution to (13) with parameter $(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$. Now we have

$$\begin{aligned}\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}) &\geq \Pi(\mathbf{x}^\star(\boldsymbol{\theta}, \mathbf{0}) + \mathbf{x}^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}}); \boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}}) \\ &= \Pi^\star(\boldsymbol{\theta}, \mathbf{0}) + \Pi^\star(\mathbf{0}, \widetilde{\boldsymbol{\theta}}),\end{aligned}$$

where the first inequality follows from the fact that the optimal value of (13) is no less than the objective value for a feasible solution. $\square$

Proof of Lemma 3. Note that $\Pi^\star(t\boldsymbol{\theta}, t\widetilde{\boldsymbol{\theta}})$ can be written as

$$\begin{aligned} t \cdot \max \quad & \sum_i p_i \frac{\lambda_i}{t} + \widetilde{p}_i \frac{\widetilde{\lambda}_i}{t} - w \sum_i \frac{\delta_i}{t} \\ \text{s.t.} \quad & \frac{\lambda_i}{t} = \theta_i \left(1 - \frac{p_i}{u}\right), \quad \forall i, \\ & \frac{\widetilde{\lambda}_i}{t} = \widetilde{\theta}_i \left(1 - \frac{\widetilde{p}_i}{\widetilde{u}}\right), \quad \forall i, \\ & \frac{\delta_i}{t} = \sum_j \left(\frac{y_{ij}}{t} - \beta \frac{y_{ji}}{t}\right) + \left(\frac{\lambda_i}{t} - \beta \sum_j \alpha_{ji} \frac{\lambda_j}{t}\right) \\ & \qquad + (1-\beta)\widetilde{\lambda}_i / t, \quad \forall i, \\ & p_i, \widetilde{p}_i, y_{ij}, \delta_i \geq 0, p_i \leq u, \widetilde{p}_i \leq \widetilde{u}, \quad \forall i, j. \end{aligned}$$

Recognizing that the optimization is (13) with parameter $(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$ under a change of variable for λ_i, $\widetilde{\lambda}_i$, δ_i and y_{ij} (which does not change the feasible set of the problem since these variables only have positivity constraints), we conclude $\Pi^\star(t\boldsymbol{\theta}, t\widetilde{\boldsymbol{\theta}}) = t\Pi^\star(\boldsymbol{\theta}, \widetilde{\boldsymbol{\theta}})$. $\square$

Proof of Theorem 1. Vectorizing, eliminating $\boldsymbol{\lambda}$, $\widetilde{\boldsymbol{\lambda}}$ and $\boldsymbol{\delta}$ using equality constraints, and relaxing the constraint $\boldsymbol{\delta} \geq \mathbf{0}$, we convert (13) into the following optimization:

$$\begin{aligned} \max_{\mathbf{p}, \widetilde{\mathbf{p}}, \mathbf{Y} \geq 0} \ & \mathbf{p}^\top \mathrm{diag}(\boldsymbol{\theta}) \Big(\mathbf{1} - \frac{1}{u}\mathbf{p}\Big) + \widetilde{\mathbf{p}}^\top \mathrm{diag}(\widetilde{\boldsymbol{\theta}}) \Big(\mathbf{1} - \frac{1}{\widetilde{u}}\widetilde{\mathbf{p}}\Big) \\ & - (1-\beta) w \boldsymbol{\theta}^\top \Big(\mathbf{1} - \frac{1}{u}\mathbf{p}\Big) - (1-\beta) w \widetilde{\boldsymbol{\theta}}^\top \Big(\mathbf{1} - \frac{1}{\widetilde{u}}\widetilde{\mathbf{p}}\Big) \\ & - (1-\beta) w \mathbf{1}^\top \mathbf{Y} \mathbf{1}, \end{aligned} \tag{17}$$

where $\mathrm{diag}(\mathbf{z})$ denotes the matrix with vector $\mathbf{z}$ on its diagonal, and we used the fact that $\mathbf{A1} = \mathbf{1}$. Since (17) is a relaxation of (13), the optimal value of (17) is an upper bound on the optimal value of (13). Problem (17) can be solved analytically. In particular, an optimal solution[6] is

$$\mathbf{p}^\star = \left(\frac{u}{2} + \frac{(1-\beta)w}{2}\right)\mathbf{1}, \quad \widetilde{\mathbf{p}}^\star = \left(\frac{\widetilde{u}}{2} + \frac{(1-\beta)w}{2}\right)\mathbf{1},$$

and $\mathbf{Y}^\star = \mathbf{0}$. For this solution of (17), if the resulting $\delta^\star$ is nonnegative, then it also constitutes a solution of (13). Note that $\boldsymbol{\delta}^\star \geq \mathbf{0}$ is equivalent to

$$\frac{u - (1-\beta)w}{u}(\mathbf{I} - \beta\mathbf{A}^\top)\boldsymbol{\theta} + \frac{\widetilde{u} - (1-\beta)w}{\widetilde{u}}(1-\beta)\widetilde{\boldsymbol{\theta}} \geq \mathbf{0},$$

which holds under Assumption 2. It follows that the solution of (17) is indeed a solution of (13). We can then recover other eliminated decision variables in (13) by substituting $\mathbf{p}^\star$, $\widetilde{\mathbf{p}}^\star$, and $\mathbf{Y}^\star$ in the equality constraints of (13). The expression of $\{c_i^\star, \widetilde{c}_i^\star\}_{i=1}^n$ is recovered based on the proof of Lemma 1. $\square$

[6] When some θ_i (or $\widetilde{\theta}_i$) is 0, any $p_i^\star$ (or $\widetilde{p}_i^\star$) in the support of $F(\cdot)$ (or $\widetilde{F}(\cdot)$) is a solution.

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